Structure of Matter - I
May 6, 2014

PROBLEM 1. On electronic structure [13 ptn]

Consider an excited Co atom ($1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^6 5d$).

a) Calculate the angle between the angular momentum vector and the z axis for a single 5d electron with m=-1. [1 ptn]
b) Sketch the radial part of this 5d wave function. [1 ptn]
c) Explain why this excited Co (…….5d) configuration is equally strong, stronger or weaker bound than a 5d electron in H, i.e., H(5d). [2 ptn]
d) After some time the Co atom has decayed to its ground electronic configuration $1s^2 2s^2 2p^6 3s^2 2p^6 4s^2 3d^7$. What is the most likely decay pathway and why? [2 ptn]

Now consider Co in its ground electronic configuration $1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^7$.

e) Determine the ground term of Co. [3 ptn]
f) Determine the ground level of Co. [1 ptn]
g) The Co atom has a nuclear spin of I=7/2. Due to the nuclear spin the ground level splits up into hyperfine levels. Determine all the possible hyperfine levels. (In case you could not determine the ground level (question 1g) then you may use J=5/2). [1 ptn]
h) What are the answers to questions 1f), 1g), and 1h) when Co is replaced by V which has not 7 but only 3 electrons in the 3d subshell? V has I=7/2 just like Co. [2 ptn]

PROBLEM 2. On nuclear structure [12 ptn]

Consider the nickel nucleus $^{51}_{28}Ni$.

a) Calculate the neutral density [in units of mass per fm^3] of the isotope. You may assume that the mass of each nucleon, no matter proton or neutron, is 1. [1 ptn]
Hint: $R_{nucl} = 1.12\, A^{1/3}$
b) Determine the nuclear spin J and the parity of this isotope. [4 ptn]
Hint: Generic sequence of nuclear shell filling: 1s, 1p, 1d, 2s, 1f, 2p, 1g, …….
c) Determine the lowest excited nuclear level? [2 ptn]

d) For nuclei the binding energy per nucleon can well be described by the semiempirical formula. For nuclei containing 51 nucleons, derive for which number of protons such nuclei have the highest binding energy per nucleon. [2 ptn]

Hint:

$$B(N,Z) = aA - bA^{2/3} - d\frac{Z^2}{A^{1/3}} - s\frac{(N-Z)^2}{A} - \frac{\delta}{A^{1/2}}$$

$a = 15.84$ MeV
$b = 18.33$ MeV
$d = 0.714$ MeV
$s = 23.20$ MeV

$$\delta = \begin{cases} +11.2 \text{ MeV} & \text{odd-odd} \\ 0 & \text{ev.-odd} \\ -11.2 & \text{ev.-ev.} \end{cases}$$

e) For the above mentioned nickel nuclei, what is the most likely decay pathway to the most stable nuclear configuration derived at d) and why? [3 ptn]

PROBLEM 3. On elementary particles [10 ptn]

Consider a bottom Λ_b baryon with quark content udb which decays into $\Lambda_c + \pi^+ + 2\pi^-$.

[quark compositions of Λ_c: udc, π^+ : $u\bar{d}$, and π^-: $d\bar{u}$]

a) Determine the charge of this Λ_b baryon? [1 ptn]
b) Determine the hypercharge of this Λ_b baryon? [1 ptn]
c) Verify that the conservation laws for lepton, charge and baryon number are respected? [1 ptn]
d) Which conservation law(s) is violated? [1 ptn]
e) By which force(s) is the decay driven? [1 ptn]
f) What is the approximate time scale of the decay? [1 ptn]
g) Consider the J=3/2 family of bottom baryons. The 6 baryons of this family have quark ompositions of: udb, usb, dsb, uub, ddb, and ssb. Determine the hypercharge Y, the azimuthal isospin I_3, and the isospin of these six baryons. [2 ptn]
h) What are the values of the color charges I_3^C and Y^C of the dsb baryon? [2 ptn]